The application of (46) gives

$$z^2 = \alpha/\beta \quad\dots\dots\dots\dots\dots\dots\dots (48),$$

and

$$\mathrm{F}(z) = \{1 + \sqrt{(\alpha\beta)}\}^2 \quad\dots\dots\dots\dots\dots (49).$$

We see then that the modulus of (43) will be a minimum, when

$$a^2 = \frac{b + 2e}{2/f + 1/b} \quad\dots\dots\dots\dots\dots\dots\dots (50),$$

and in taking the square root the ambiguity must be so determined as to make the real part of a positive.

Equation (50) coincides with that obtained by Mr. Heaviside for the case where all the quantities are real.

II. "On the Influence of Pressure on the Spectra of Flames." By G. D. LIVEING, M.A., F.R.S., Professor of Chemistry, and J. DEWAR, M.A., F.R.S., Jacksonian Professor, University of Cambridge. Received January 22, 1891.

We have already described ('Phil. Trans.,' A, 1888) the remarkable spectrum of the oxy-hydrogen flame burning at the ordinary atmospheric pressure. Recently we have examined the spectrum of the same flame at various pressures: hydrogen burning in excess of oxygen up to a pressure of 40 atmospheres, and oxygen in excess of hydrogen up to a pressure of 25 atmospheres, also that of the mixed gases burning in carbonic acid gas.

The apparatus employed was an adaptation of one of the tubes used in our experiments on the absorption spectra of compressed gases ('Phil. Mag.,' September, 1888, and 'Roy. Soc. Proc.,' vol. 46, p. 222). It consisted of a steel cylinder, about 50 mm. in internal diameter and 225 mm. long, fitted at one end with a quartz stopper, a, in the annexed figure, and with a jet, b, for burning the gas, adapted by a properly fitting union joint to the opposite end. There were two tubes, c and d, connected to the cylinder at the sides, of which one, c, served for the introduction of gas, while the other, d, was fitted with a stopcock and was used to draw off the water formed, or to reduce the pressure of the gas in the cylinder if that was desired. The flame was observed, nearly end on, through the quartz stopper. The whole apparatus was kept cool by a stream of cold water running on to a sponge cloth wrapped round the cylinder. In the course of the tube conveying gas to the jet b was interposed a small cylinder, e, in which sodium was placed, and by heating this, the gas entering could be charged with sodium vapour.

Q 2

The gases were supplied from steel cylinders into which they had been compressed, and the pressure was registered by a gauge attached to the tube by which the gas entered the experimental cylinder. Commercial compressed gases were used, containing a sensible percentage of air.

When hydrogen was the gas forming the burning jet, it was lighted at the end of the tube *b* before introducing it into the experimental cylinder. When it was desired to have a jet of oxygen burning in hydrogen, this could be managed by introducing oxygen through the second tube and increasing the supply of hydrogen until the flame passed over to the oxygen jet. The same result was sometimes attained by first filling the experimental cylinder by a gentle stream of hydrogen through the side tube *c* before the end with the tube *b* was screwed on ; the hydrogen as it issued was then lighted, and the jet, with a gentle stream of oxygen issuing, inserted and screwed down. The stopcock *s* was kept open until this was done, and then by closing *s*, and admitting more gas from the reservoirs, the pressure in the experimental cylinder could be increased at pleasure.
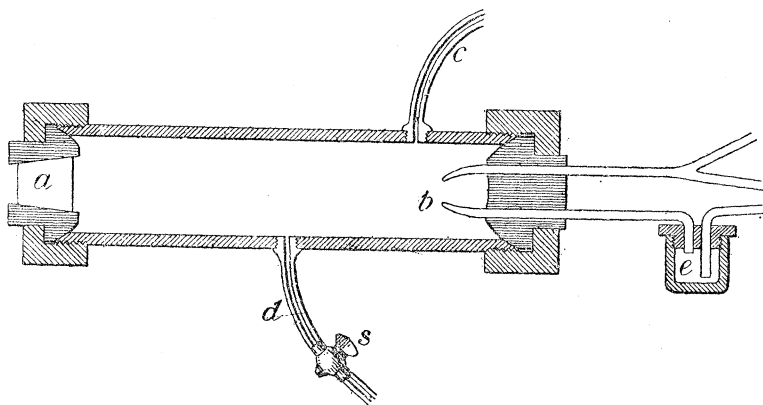
Hydrogen Burning in Oxygen.

The first observations were made with a jet of hydrogen burning in oxygen. As the pressure rose, the luminosity of the flame increased, as long ago described by Frankland ('Experimental Researches,' p. 905). The colour of the flame, viewed end on, was yellow, as if it contained sodium ; but, on examining it with a spectroscope, it was found to give a continuous spectrum intersected by many shaded bands, and the D lines of sodium were only faintly present. The shaded bands were faint at a pressure of 5 atmospheres, but at pressures of 20 atmospheres and upwards they came out strongly. They were evidently the absorption bands of NO_2,

derived from the residue of atmospheric air mixed with the condensed gases. We took a photograph of them, and on comparing this with a photograph of the NO_2 bands, we found the two to be identical. Except for the bands, and the bright lines of sodium, the spectrum appeared to be continuous, and to extend from about λ 6200 to λ 4150, with the brightest part about λ 5150. It increased in brilliance as the pressure increased, as well as in extent, being visible at 3 atmospheres pressure from about λ 6720 to λ 4040. The greater distinctness of the NO_2 bands at the higher pressures was due both to the greater brightness of the continuous spectrum and to the greater quantity of NO_2 formed. A large quantity of water accumulated in the experimental tube, and when this was drawn off by the stopcock s, it effervesced with escape of NO, and was found to be strongly acid. A specimen titrated was found to contain very nearly 3 per cent. of nitric acid. The observations were continued up to a pressure of 40 atmospheres. There was no indication that the continuous spectrum had any connexion with the line spectrum of hydrogen. There was no increase of brilliance in the neighbourhood of the C, F, or G lines of hydrogen. The characters of the spectrum were, however, better seen in the absence of NO_2, and will be described in the next section.

Oxygen Burning in Hydrogen.

In this case the colour of the flame was very different from that of hydrogen burning in oxygen. Instead of being yellow, it appeared, to the unaided eye, to have a lavender hue. In the spectroscope it showed a perfectly continuous spectrum, brightest in the green, about the region of the Fraunhofer line b, and very gradually fading away on either side. On the red side it could be just traced up to about λ 6150, and on the violet side to about λ 4285, at ordinary pressures. The sodium lines were absent. With increase of pressure it increased very much in brightness, and at 8 atmospheres pressure it could be traced as low as λ 6630 and as high as λ 3990.

The dispersion used was that of a direct-vision spectroscope (such as was described by us, 'Roy. Soc. Proc.,' vol. 41, p. 449), equivalent to three prisms of white flint glass, but the collimator and telescope very short, so as to obtain plenty of light. With less dispersion, perhaps, the continuous spectrum might have been traced further. Photographs, however, showed that it scarcely extended into the ultra-violet. There was no indication that this spectrum was due to an expansion of the lines of either the first, or second, spectrum of hydrogen. It is true that the maximum brightness (which could not be determined with any great accuracy) was not very far from F, but no indication of any second maximum in the neighbourhood of either

C or G, or anywhere else, could be detected. The pressure was carried up to 12 atmospheres, and at this pressure the visible spectrum was brilliant, but, in the ultra-violet, photographs showed that the spectrum consisted only of what we have called the " water-spectrum," very strong and sharp. The lines of this spectrum showed no signs of expansion even at a pressure of 12 atmospheres, and, though much more intense than at ordinary pressures, remained clearly defined.

Observations were continued with the eye up to 25 atmospheres pressure, but no trace of emission, or absorption, corresponding to either spectrum of hydrogen could be detected, and it is doubtful if either spectrum can be produced in such a flame. Since the formation of steam from its component gases is attended with a diminution of volume, increased pressure will increase the stability of the compound, and the flame will contain a larger proportion of steam, as well as have a higher temperature, than at ordinary pressures.

The water formed when the flame was a jet of oxygen burning in hydrogen was found to be alkaline, and to contain ammonia. But the proportion of ammonia was much less than the proportion of nitric acid formed when the jet was hydrogen burning in oxygen; a specimen titrated contained 0·004 per cent. of ammonia.

Effects of Pressure on the Sodium Spectrum.

In order to see what effect would be produced by increased pressure on the spectrum of other substances in the flame, we charged the hydrogen with sodium vapour by making it pass, before entering the experimental cylinder, through a small iron cylinder, *e* in the figure, containing metallic sodium, heated by a lamp. As the D lines of sodium are very easily expanded and self-reversed in a flame at ordinary pressure, some care was needed to discriminate the effects which were really to be ascribed to pressure. The gas was easily charged with sodium vapour, and when burning in oxygen, not only the D lines, but the citron and green pairs, and sometimes the blue pair (λ 467), and the orange pair (λ 616), were well seen; but we could not find that they were expanded by increase of pressure. A sudden change of pressure generally produced an expansion, but it did not last; the lines fined down again when the pressure was steady, whether that pressure was high or low. These experiments were continued up to a pressure of 40 atmospheres without any definite effect on the width of the lines which could be ascribed to the pressure.

It may be said that at the higher pressure the evaporation of the sodium would be slower, and so the proportion of sodium vapour to hydrogen be diminished; also, when the lines are diffuse at the

edges to begin with, it is extremely difficult to judge whether there is any expansion. At all events, we may say that there is no expansion produced by pressure at all comparable with that produced in a flame at ordinary pressure by increasing the quantity of sodium in the flame. We noticed, however, that the · presence of sodium, which produces a feeble continuous spectrum in a flame at an ordinary pressure, seemed to increase the continuous spectrum of the flame under pressure, especially in the orange and green.

Oxy-hydrogen Jet in Carbonic Acid Gas.

For this experiment a two-branched tube (the upper one in the figure) was used. The jet of mixed oxygen and hydrogen was first lighted and introduced into the experimental cylinder while the latter was full of air and the stopcock *s* open. The air was then replaced by CO_2 entering by the tube *c*. The effect of this was at once to brighten the flame and change its colour from yellow to blue. Seen in the spectroscope, the change consisted in an increase of continuous spectrum, especially towards the more refrangible end. When the stopcock *s* was closed so that the pressure rose in the experimental cylinder, the flame increased in brightness, but there was no other change in the spectrum. It remained continuous with no bright or dark lines, or bands, except the D lines of sodium. It resembled an ordinary flame of CO. The jet would not burn in CO_2 unless there was some excess of oxygen, and even with an excess of oxygen we could not get it to continue to burn in CO_2 at a pressure higher than 2 atmospheres.

Ethylene in Oxygen.

A jet of ethylene burning in oxygen gave, when the flame was small, the usual candle-flame spectrum, together with a band in the indigo (λ 431) shading towards the violet; but as the pressure was increased the continuous spectrum brightened and completely over-powered the bands, and at the same time the absorption spectrum of NO_2 appeared. We carried the pressure up to 33 atmospheres, and at that pressure the flame seemed to give nothing but a continuous spectrum, intersected by the absorption bands of NO_2. In our tube, the flame was viewed almost directly end on, and it is possible that if we had seen the flame sideways, we might have detected the hydro-carbon flame spectrum near the nozzle. At the high pressure much soot separated. We tried burning a mixture of ethylene and oxygen. The mixed jet burnt well in air and, when the supply of oxygen was sufficient, gave the hydrocarbon flame spectrum. In the experi-mental tube in oxygen, the jet burnt well at the atmospheric pressure,

but we failed to get it to continue burning when the pressure was increased. The shaded band, commencing with a sharply-defined edge about λ 431, seems to be independent of the pressure, and has been before observed in a gas flame (Huggins, 'Roy. Soc. Proc.,' vol. 30, p. 580). In fact, the only effect of pressure in this, as in the former cases, seemed to be the increase of the continuous spectrum.

Cyanogen and Oxygen.

As we could not obtain cyanogen at such pressures as we had used in the case of the other gases, we were obliged to content ourselves with exploding mixtures of cyanogen and oxygen in an iron bottle, fitted with a quartz stopper like that of the experimental tube above described. The bottle, having been exhausted by an air-pump, was filled with the mixture of gases, and exploded by an electric spark. With less than 3 vols. of oxygen to 1 vol. of cyanogen, there was always a considerable deposit of carbon, which covered the quartz and impeded vision; but, with 3 vols. of oxygen to 1 of cyanogen, the carbon was all burnt. Notwithstanding the brilliant banded spectrum of a flame of cyanogen in oxygen at ordinary pressure, nothing but a continuous spectrum could be seen in the flash of the exploded gases, except the ubiquitous D lines of sodium. The continuous spectrum was bright. Photographs showed a continuous spectrum with lines of iron, calcium, potassium, and sodium, but no cyanogen or carbon bands, or carbon lines. When a little hydrogen was added to the mixture of gases, no trace of the hydrogen red or green line could be detected in the spectrum of the exploding gas.

In every case, the prominent feature of the light emitted by flames at high pressure appears to be a strong continuous spectrum. There is not the slightest indication that this continuous spectrum is produced by the widening of the lines, or obliteration of the in-equalities, of the discontinuous spectra produced by the same gases at lower pressures. On the contrary, it seems to be developed independently. This is, on the whole, quite in accordance with what would be expected, considering that under pressure the molecules of the gases have much less freedom, encounters amongst them are much more frequent, and they have much less chance of vibrating independently, and of taking up exclusively, or chiefly, the fundamental rates of vibration which are natural to them when free. Their condition, during a large part of any given time, approximates to that of the molecules of a liquid, and their spectra approximate to that of a liquid to at least a like extent. On the other hand, the higher temperature which, in many flames, attends an increased pressure ought to give some intensity to the special radiation which the molecules emit during their time of free motion ; and this we have

noticed to occur in the principal sections of the discontinuous spectrum of the oxy-hydrogen flame. Whether the continuous spectrum is due to the mutual action of the molecules of the compressed gases may perhaps be best determined by some photometric measures of the rate at which the brilliance increases with the pressure. Frankland ('Exp. Researches,' pp. 892 *et seq.*) has made some such measures, but not sufficient to solve the question. We have made an attempt to measure, not the total intensity of the light, but that of rays of definite refrangibility.

Photometry of Oxy-Hydrogen Flame under Pressure.

The apparatus used for these measures was a spectro-photometer of the pattern employed by Crova ('Annales de Chimie,' ser. 5, vol. 29, p. 556). In this, the rays of one of the sources of light to be compared are passed through two Nicol's prisms, and then reflected into one half of the slit of the spectroscope, while the light from the other source passes directly into the other half of the slit. By turning one of the Nicol prisms, the light from the first source can be reduced at pleasure, and any small section of the spectrum can be separately observed by cutting off the rest by means of a shutter in the eye-piece. We found it by no means easy to get good concordant observations. A much larger vessel was used than for the earlier experiments, one which contained several litres, and so we may presume a more uniform pressure was maintained within it. The results of the best series of observations on the photometric intensity of the jet of oxygen burning in hydrogen are given in the following table. The comparison light was a petroleum lamp.

1.	2.	3.	4.
15 lbs.	3°	274	$30 \times 3^2 = 270$
35	7	1485	$30 \times 7^2 = 1470$
55	11	3641	$30 \times 11^2 = 3630$
75	14	5853	$26 \times 15^2 = 5850$
95	19	10600	$29 \times 19^2 = 10469$

The first column gives the pressure of the gas, the second the mean of four to six observations of the angular deviation of the Nicol's prisms from the position of complete extinction, for each pressure. The third column gives the squares of the sines of the angles in the second column multiplied by 100,000.

It will be seen from the last column that the numbers in the third column, which should be proportional to the photometric intensities

at the respective pressures, are approximately proportional to the squares of the pressures.

This may be taken to indicate that the brightness of the continuous spectrum depends mainly on the mutual action of the molecules of gas.

A series of similar observations on hydrogen burning in oxygen gave somewhat different results, tabulated below :—

1.	2.	3.
15 lbs.	6°	1093
35	13	5060
55	18	9549
75	22	14033
95	28	17861

The flame was brighter than that of oxygen burning in hydrogen at ordinary pressure, but the rate of increase with increased pressure was not so rapid as in the former case. It seems as if the continuous spectrum were made up of two parts, one varying as the square of the pressure, and another according to some other law. The flame is evidently not the same in the two cases. The products of combustion derived from the small quantity of air are different, and also the hydrogen jet always showed the presence of sodium, sometimes calcium. The appearance of the flame was also different; the hydrogen jet being faintly visible and yellowish in the elongated part, whereas the light from the oxygen jet was concentrated near the base, the point being invisible. The measures of which the means are tabulated above were also less concordant than the corresponding measures for the oxygen jet. We were unable to carry our measures beyond a pressure of 95 lbs., because at higher pressures a cloud was formed in the apparatus which prevented our seeing the flame directly. We hope to prosecute these measures with flames of other gases, and, if possible, at higher pressures.

The conclusions to which our experiments have led seem inconsistent with those which have been drawn from Plücker and Hittorf's well-known observations on the widening of the hydrogen lines in vacuous tubes with a residue of hydrogen when that residue increases. That the widening of the lines in a Plücker's tube results from increasing the density of the residue of hydrogen in the tube cannot be gainsaid, but we are wholly ignorant of the mechanism by which the gas is lighted up by the electric discharge. It is sometimes assumed, but without any sufficient reason, that the energy of the electric current is first converted into heat, and then in turn into

radiation ; but the electric energy may equally well be directly con-
verted into the motion of radiation.   As a fact, we have never yet
been able to obtain either the emission or the absorption spectrum of
hydrogen without the aid of an electric current, so that, in reasoning
on this spectrum, we are much more in a region of speculation than
when treating of flames.   Whether the hydrogen lines, bright or
dark, in the solar spectrum are produced directly by the high
temperature of the sun, may even be called in question.   And though
we may admit that the density of the hydrogen in the sun's atmo-
sphere, outside the photosphere, is but slight, it does not follow that
the total pressure of all the gases forming that atmosphere is so very
small as Messrs. Frankland and Lockyer (' Roy. Soc. Proc.,' vol. 17,
p. 288) have, from the width of the lines, concluded it to be.   After
all, it is not so easy to connect the temperature, even of a flame,
with its radiation, for it is only when the condition of a gas is steady
that we can assume that there is a definite relation between the
motion of agitation, on which temperature depends, and the vibratory
motions, on which radiation depends.   In speculating on such
questions, chemical, as well as electrical, changes must not be lost
sight of, although the latter may be more directly concerned in
radiation.

Experiments which we have commenced upon the arc in an
atmosphere of compressed gas tend to the same conclusion.   It does
not appear that the metallic lines in the arc are sensibly affected by
a steady pressure up to 15 atmospheres.   The details of these observa-
tions, which are complicated by the variation of resistance with
change of pressure, we defer until the experiments are finished.

III. " On the Focometry of Lenses and Lens-Combinations, and
on a new Focometer."   By SILVANUS P. THOMPSON, D.Sc.,
B.A., Professor of Physics in the City and Guilds Technical
College, Finsbury.   Communicated by Professor G. CAREY
FOSTER, B.A., B.Sc., F.R.S.   Received February 4, 1891.

(Abstract.)

Few of the accepted methods of focometry take into account the
distance between the two principal points (or Gauss points) of a lens,
or afford the means of measuring this distance, as well as the true
focal length, and some of them are open to the objection that they
necessitate troublesome double adjustments.   Of these methods the
author gives a brief categorical review.

He has devised a method in which there are no double adjustments,
no measurements of the size of optical images, no assumptions as to